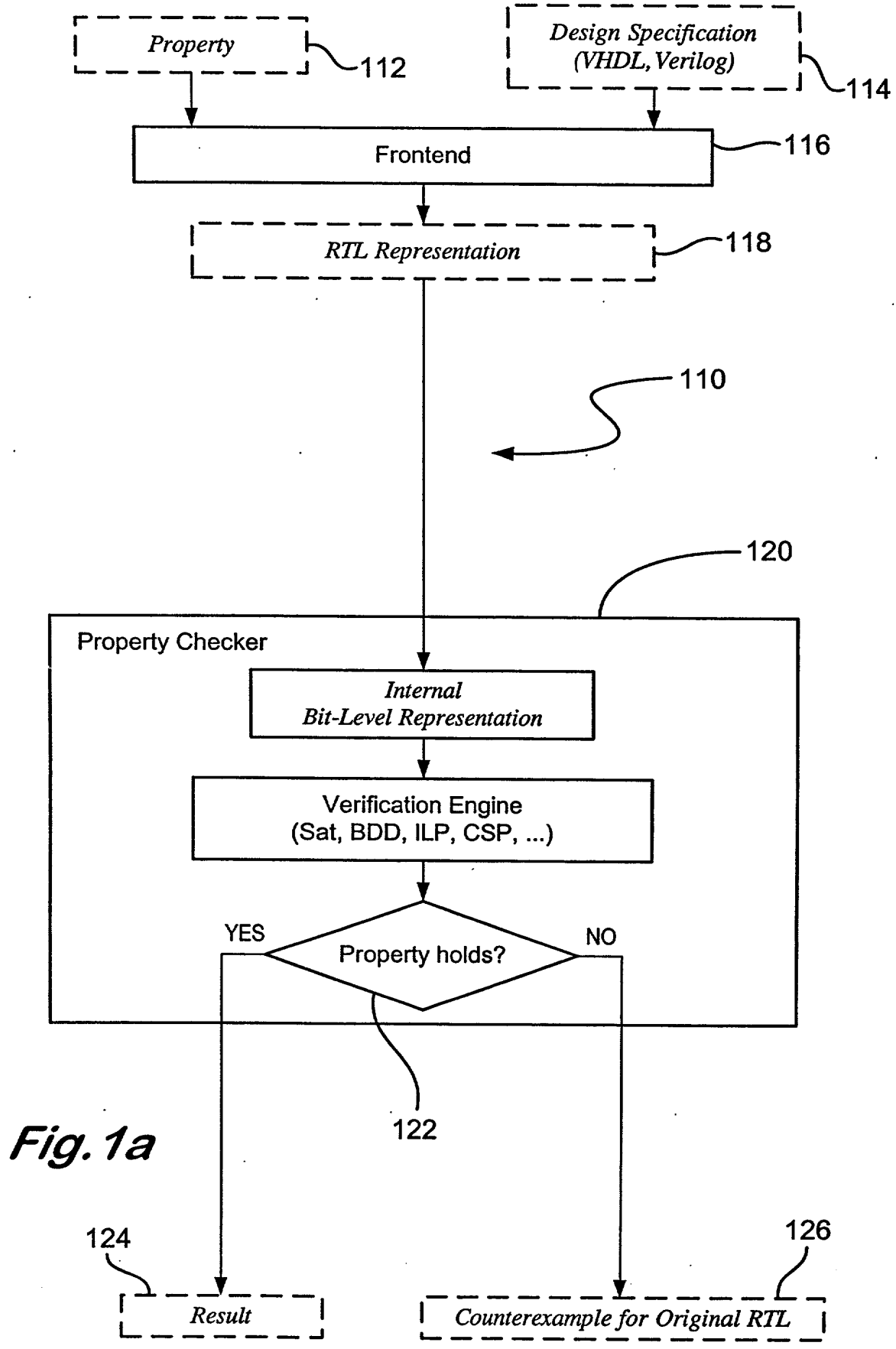


Fig. 1a

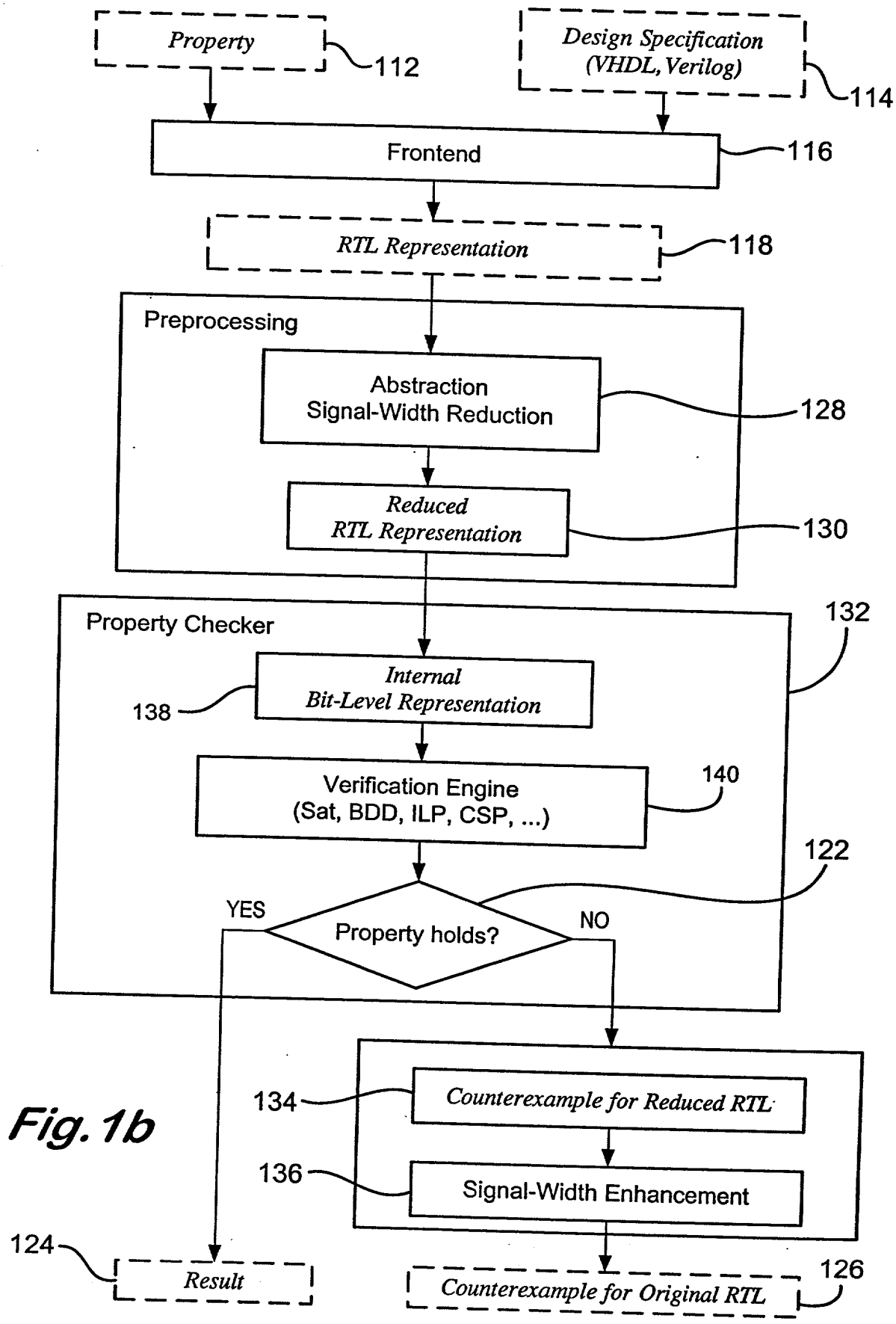
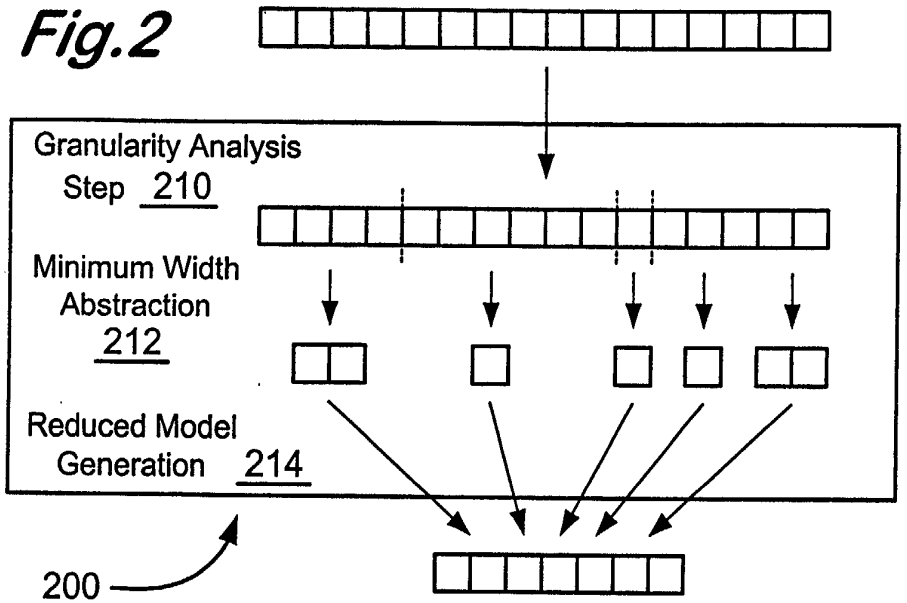


Fig. 1b

Fig.2



Original Signal $x_{[16]}$

216

Chunks : $x_{[16]}[15,12], x_{[16]}[11,6], x_{[16]}[5,5], x_{[16]}[4,2], x_{[16]}[1,0]$

218

Minimum Widths: 2 + 1 + 1 + 1 + 2 = 7

220

Corresponding Reduced Signal $x'_{[7]}$

222

Fig.4

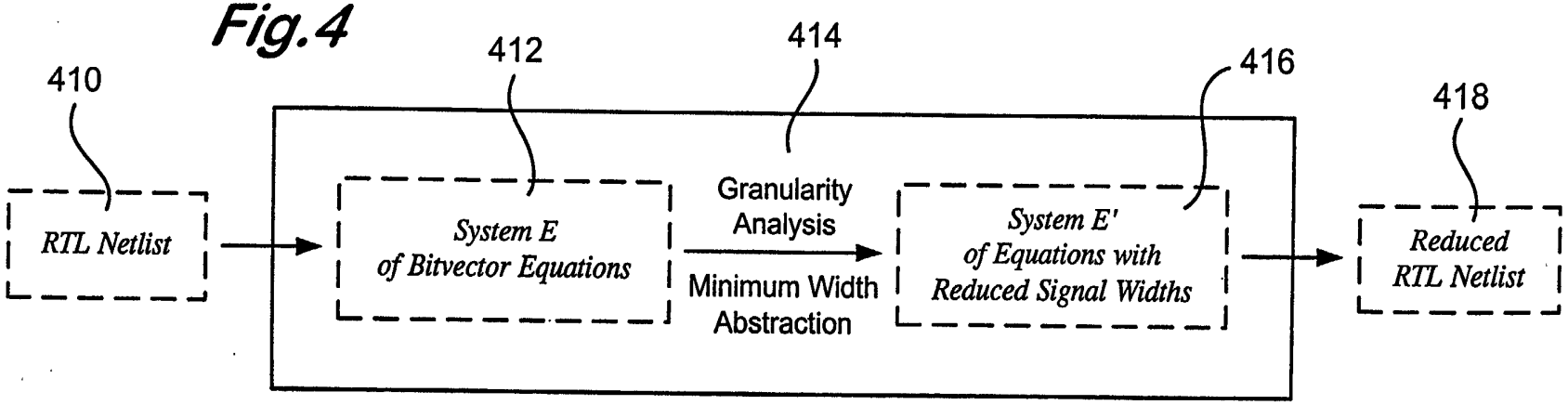


Fig.3

Bitvector Operator	Syntax	Example
bitvector variables bitvector constants	$x_{[n]}$ $c_{[m]}$	$x_{[8]}, y_{[1]}, z_{[4]}, \ldots$ $10011_{[5]}, 00111111_{[8]}, 0_{[1]}, 1_{[1]}, \ldots$
concatenation	$\otimes$	$x_{[16]} \otimes y_{[4]}$
extraction	$[j, i]$	$x_{[8]}[5, 2]$
bitwise negation (inversion)		neg $(x_{[8]})$
bitwise Boolean operations	and, or, xor nand, nor, xnor	$x_{[12]}$ and $y_{[12]}$, $x_{[12]}$ or $y_{[12]}$, $x_{[12]}$ xor $y_{[12]}$ $x_{[12]}$ nand $y_{[12]}$, $x_{[12]}$ nor $y_{[12]}$, $x_{[12]}$ xnor $y_{[12]}$
if-then-else	ite	ite$(a_{[4]} = b_{[4]}, x_{[8]}, y_{[8]})$ ite$(a_{[4]} < b_{[4]}, x_{[8]}, y_{[8]})$
arithmetic	$+, -$ $*$	$x_{[32]} + y_{[32]}, x_{[32]} - y_{[32]}$ $x_{[16]} * y_{[16]}$
memory read memory write	$mem_{[m \cdot n]}[i_{[l]}]$	$x_{[10]} := mem_{[128 \cdot 10]}[i_{[7]}]$ $mem_{[32 \cdot 8]}[i_{[5]}] := x_{[8]}$

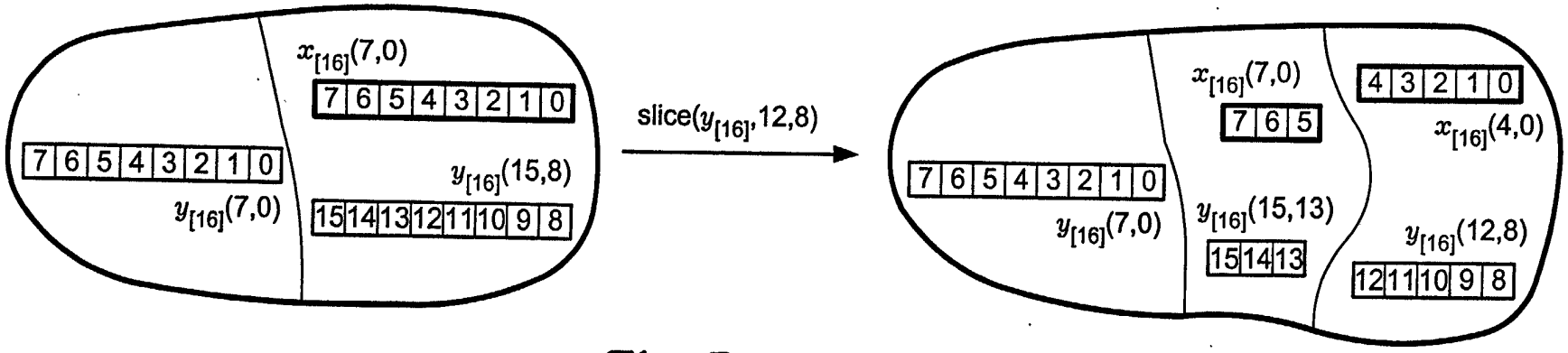

$x_{[16]}(7,0)$
7 6 5 4 3 2 1 0
7 6 5 4 3 2 1 0
$y_{[16]}(7,0)$
$y_{[16]}(15,8)$
15 14 13 12 11 10 9 8
slice($y_{[16]}$,12,8)
$x_{[16]}(7,0)$
7 6 5
4 3 2 1 0
$x_{[16]}(4,0)$
7 6 5 4 3 2 1 0
$y_{[16]}(7,0)$
$y_{[16]}(15,13)$
15 14 13
$y_{[16]}(12,8)$
12 11 10 9 8

Fig.5

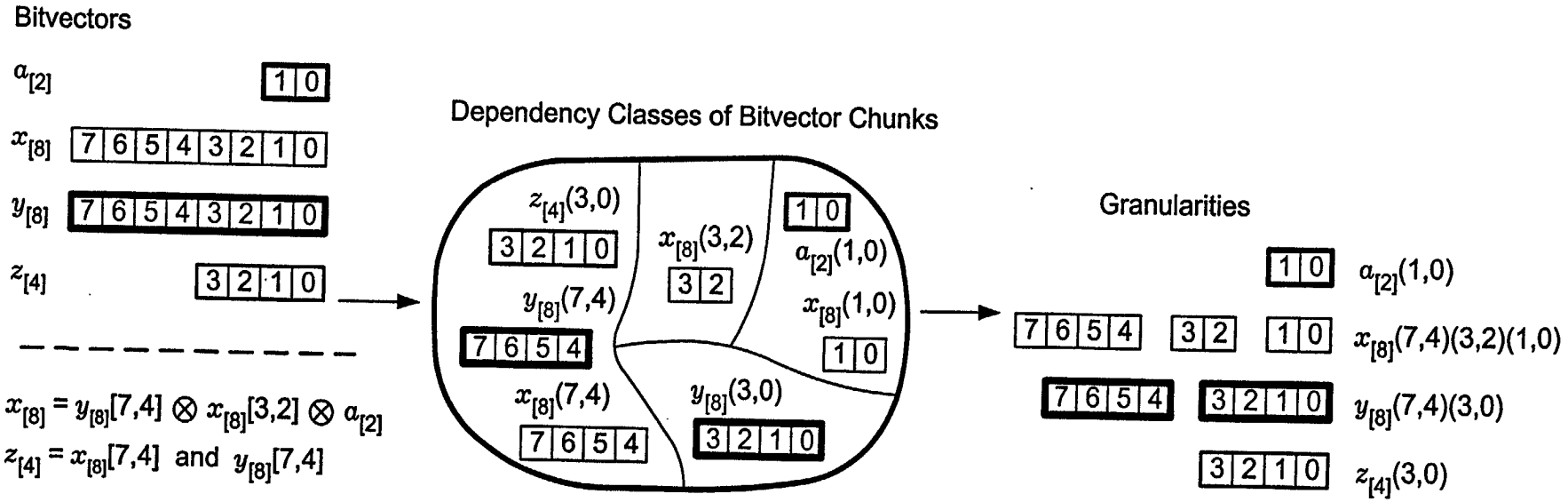

Bitvectors
$a_{[2]}$ 1 0
$x_{[8]}$ 7 6 5 4 3 2 1 0
$y_{[8]}$ 7 6 5 4 3 2 1 0
$z_{[4]}$ 3 2 1 0
$x_{[8]} = y_{[8]}[7,4] \otimes x_{[8]}[3,2] \otimes a_{[2]}$
$z_{[4]} = x_{[8]}[7,4]$ and $y_{[8]}[7,4]$
Dependency Classes of Bitvector Chunks
$z_{[4]}(3,0)$
3 2 1 0
$y_{[8]}(7,4)$
7 6 5 4
$x_{[8]}(7,4)$
7 6 5 4
$x_{[8]}(3,2)$
3 2
$y_{[8]}(3,0)$
3 2 1 0
1 0
$a_{[2]}(1,0)$
$x_{[8]}(1,0)$
1 0
Granularities
1 0 $a_{[2]}(1,0)$
7 6 5 4 3 2 1 0 $x_{[8]}(7,4)(3,2)(1,0)$
7 6 5 4 3 2 1 0 $y_{[8]}(7,4)(3,0)$
3 2 1 0 $z_{[4]}(3,0)$

Fig.6

Fig.6a

Process 1 Granularity Analysis of Bitvector Equations

```
1  gran(e) {
2    switch (e);
3    case e ≡ 's[n] = t1[m1] ⊗ t2[m2]':
4    case e ≡ 's[n] = (t1[m1] ⊗ t2[m2])[j, i]':
5
6
7
8
9
10
11   case e ≡ 's[n] = (t[m][l,k])[j,i]':
12   case e ≡ 's[n] = ite(a[m] = b[m], t[t[n]], te[n])[j,i]':
13   case e ≡ 's[n] = (t1[m] and t2[m])[j, i]':
14   ...
15   case e ≡ 's[n] = neg(t[n])':
16   case e ≡ 's[n] = ite(a[m] = b[m], t[t[n]], t[e[n]])':
17   case e ≡ 's[n] = t1[n] and t2[n]':
18   ...
19   case e ≡ 'x[n][j,i] = y[m][l,k]':
20 }
```

```
gran('s[n][n−1, m2] = t1[m1]'); gran('s[n] [m2−1,0] = t2[m2]');
if (j<m2) {
 gran('s[n] = t2[m2][j, i]');
} else if (i ≥ m2) {
 gran('s[n] = t1[m1][j−m2, i−m2]');
} else {
 gran('s[n] = t1[m1][j−m2, 0] ⊗ t2[m2][m2−1, i]');
}
gran('s[n] = t[m][k + j, k + i]');
gran('s[n] = ite(a[m] = b[m], t[t[n]][j,i], t[e[n]][j,i])');
gran('s[n] = t1[m][j,i] and t2[m][j, i]');

gran('s[n] = t[n]');
gran('a[m] = b[m]'); gran('s[n] = t[t[n]]'); gran('s[n] = t[e[n]]');
gran('s[n] = t1[n]'); gran('s[n] = t2[n]');

slice(x[n], j,i); slice(y[m], l,k); join(x[n]⟨j,i⟩, y[m]⟨l,k⟩);
```

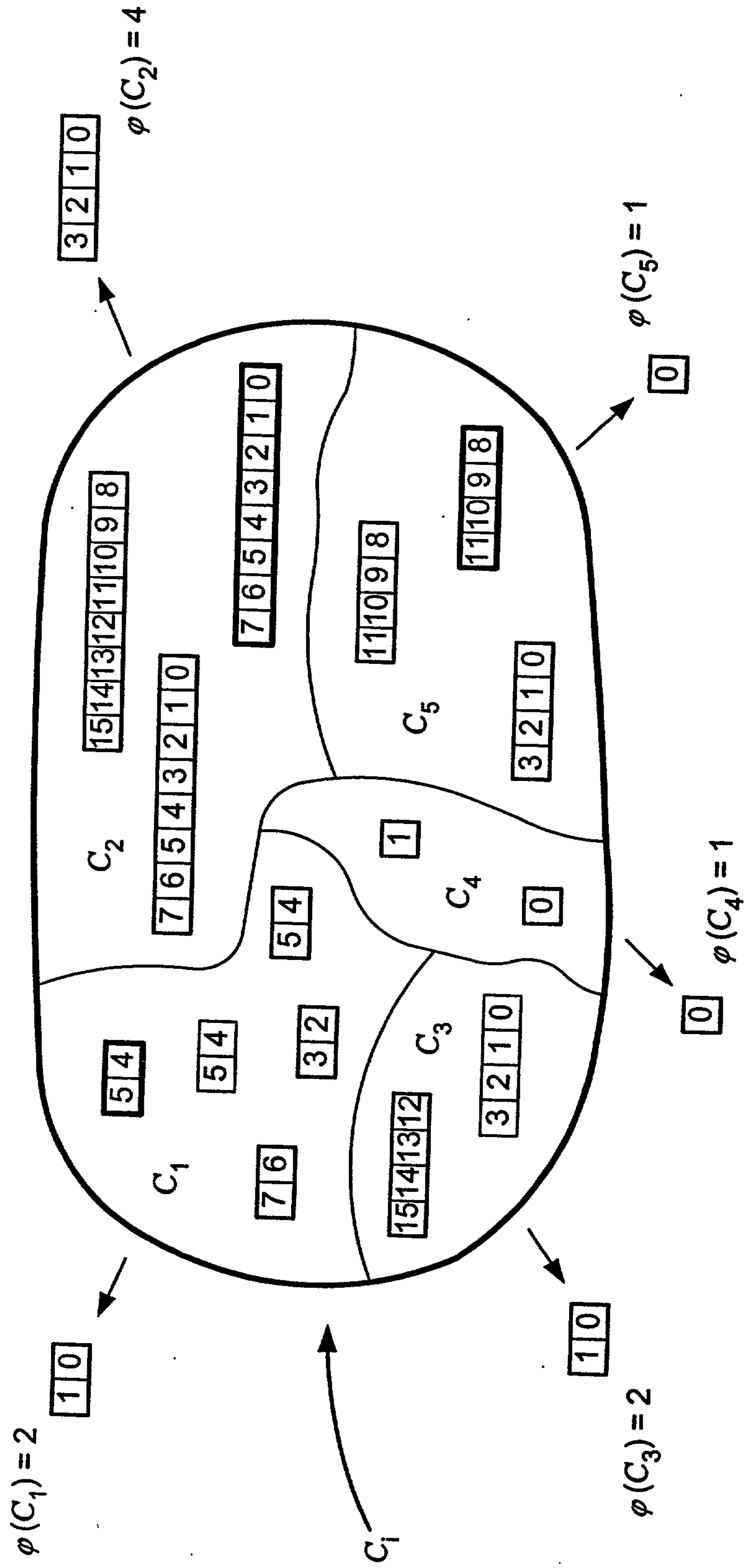

φ(C2) = 4
3 2 1 0
φ(C5) = 1
0
φ(C4) = 1
0
φ(C3) = 2
1 0
φ(C1) = 2
1 0
Ci
C1
C2
C3
C4
C5
15 14 13 12 11 10 9 8
7 6 5 4 3 2 1 0
11 10 9 8
3 2 1 0
5 4
7 6
3 2
15 14 13 12
1
0

Fig. 7

Fig. 7a

Process 2 Reduced Model Generation

```
1  for each bitvector variable x_[n] {
2    m := 0 ;
3    for each chunk x_[n]⟨j, i⟩ of the computed granularity of x_[n] {
4      C := find ( x_[n]⟨ j, i ⟩ ) ;   // equivalence class containing x_[n]⟨j,i⟩
5      m := m + φ(C) ;
6    }
7    if (m ≥ n) then m := n ;
8    replace all occurrences of x_[n] in the bitvector equations by x'_[m] ;
9    and adjust all extraction expressions affected by x_[n] ;
10 }
```

Fig.8

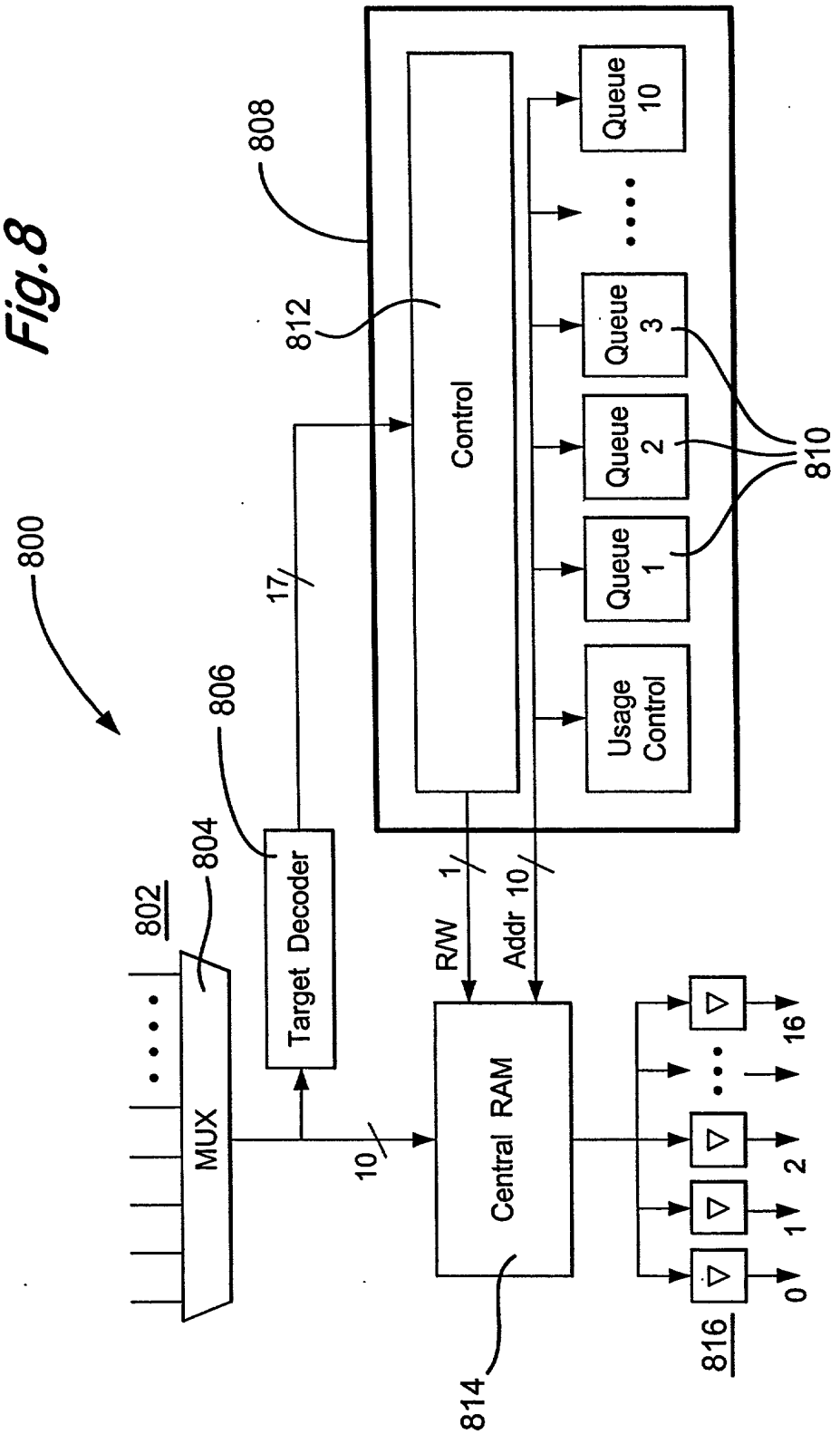
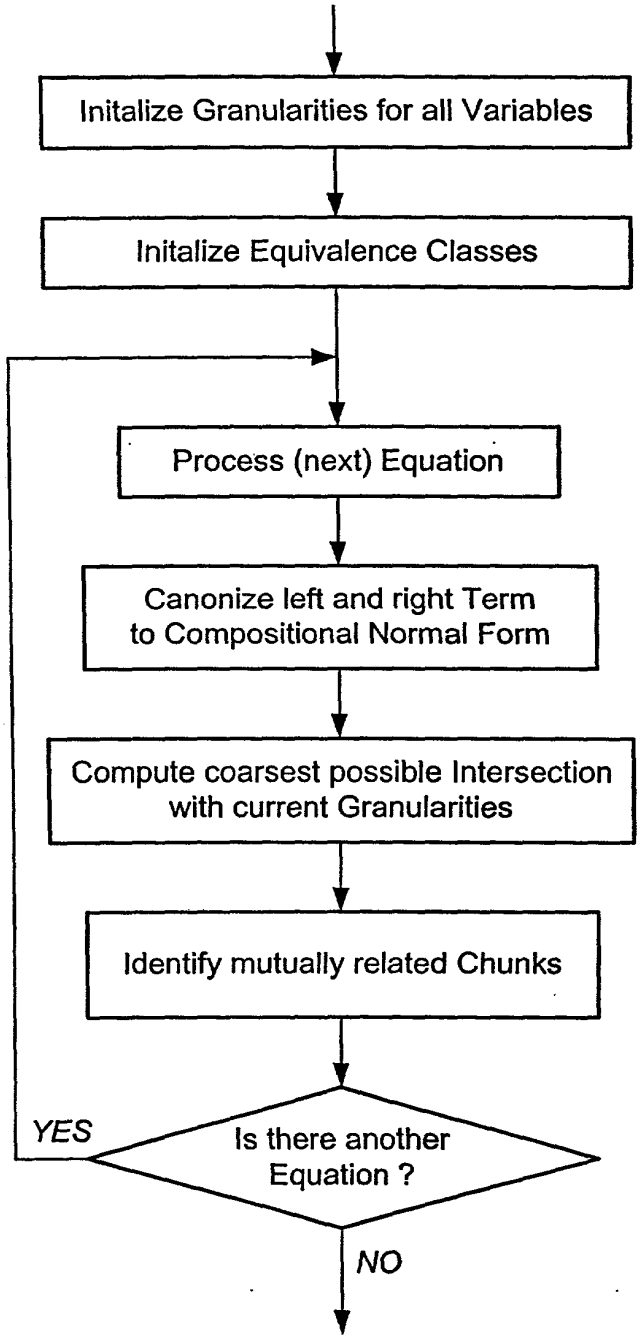


Fig.9

	Property	Original design	Reduced model
Computation times of the prototype for analysis and reduced model generation	nop read write		2.96 secs 6.53 secs 3.24 secs
FIFO sizes on RTL	nop read write	160 cells x 10 bit 160 cells x 10 bit 160 cells x 10 bit	160 cells x 2 bit 160 cells x 3 bit 160 cells x 3 bit
Overall number of bits in all relevant signals (cones of influence of the property)	nop read write	20925 31452 14622	5034 (24.0 %) 10592 (33.6 %) 5163 (35.3%)
Overall number of gates in synthesized netlist	nop read write	23801 23801 23801	5661 (27.9 %) 7929 (33.3 %) 7929 (33.3 %)
Number of state bits	nop read write	1658 1658 1658	362 (21.8 %) 524 (31.6 %) 524 (31.6 %)
Property checker runtimes	nop read write_fail write_hold	23:33 min 42:23 min 2:08 min 27:08 min	37.96 secs (2.7 %) 3:27 min (8.1 %) 25.66 secs (19 .5 %) 1:08 min (4.2 %)



$x_{[8]}[7,2] \otimes x_{[8]}[1,0] = (a_{[4]} \otimes b_{[4]} \otimes c_{[4]})[11,4]$

$x_{[8]}[7,0] = a_{[4]}[3,0] \otimes b_{[4]}[3,0]$

$x_{[8]}[7,4] \otimes x_{[8]}[3,0] = (a_{[4]}[3,0] \otimes b_{[4]}[3,0]$

$x_{[8]}[7,4] \longleftrightarrow a_{[4]}[3,0]$
$x_{[8]}[3,0] \longleftrightarrow b_{[4]}[3,0]$

Fig. 10

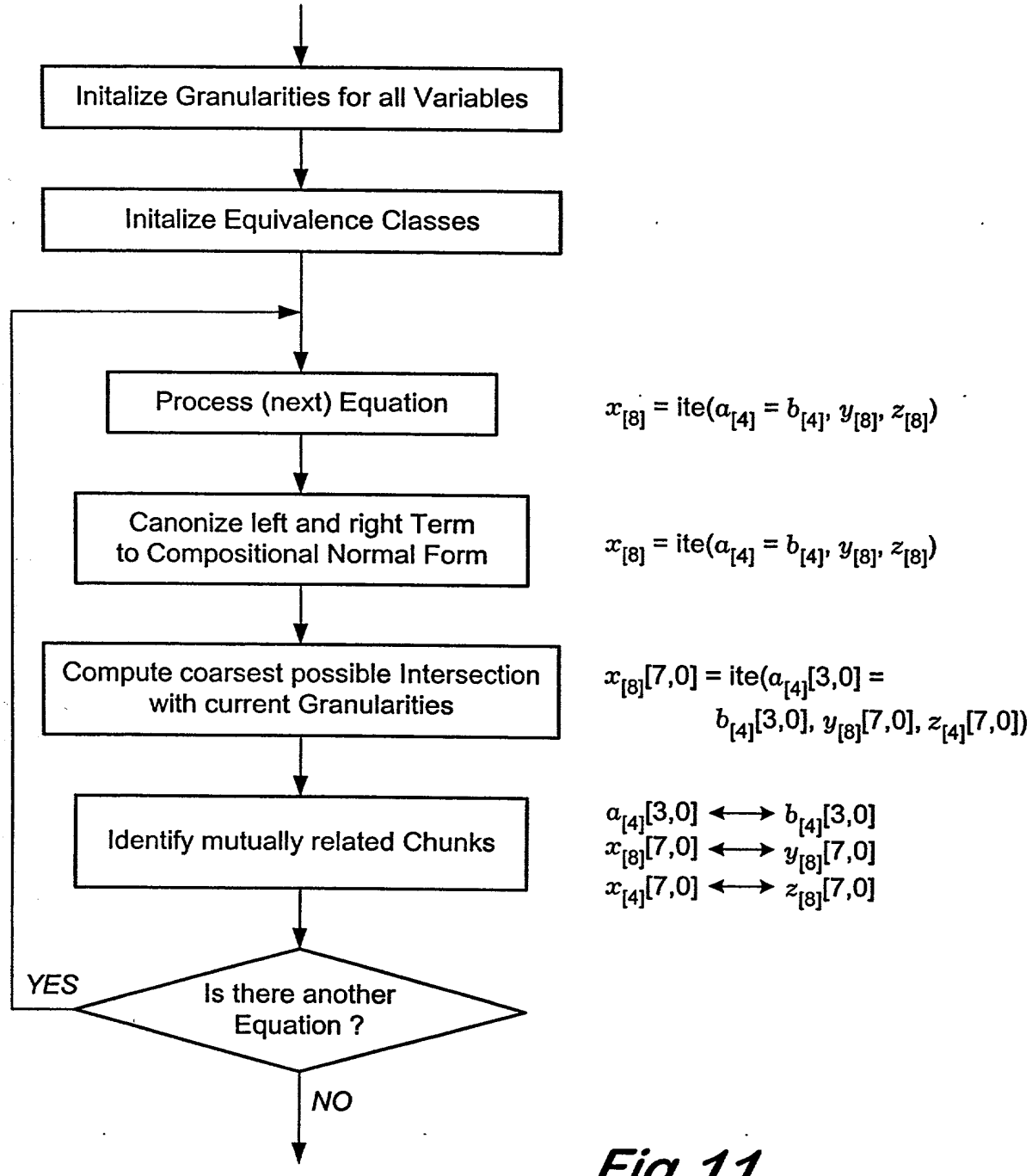


Fig. 11

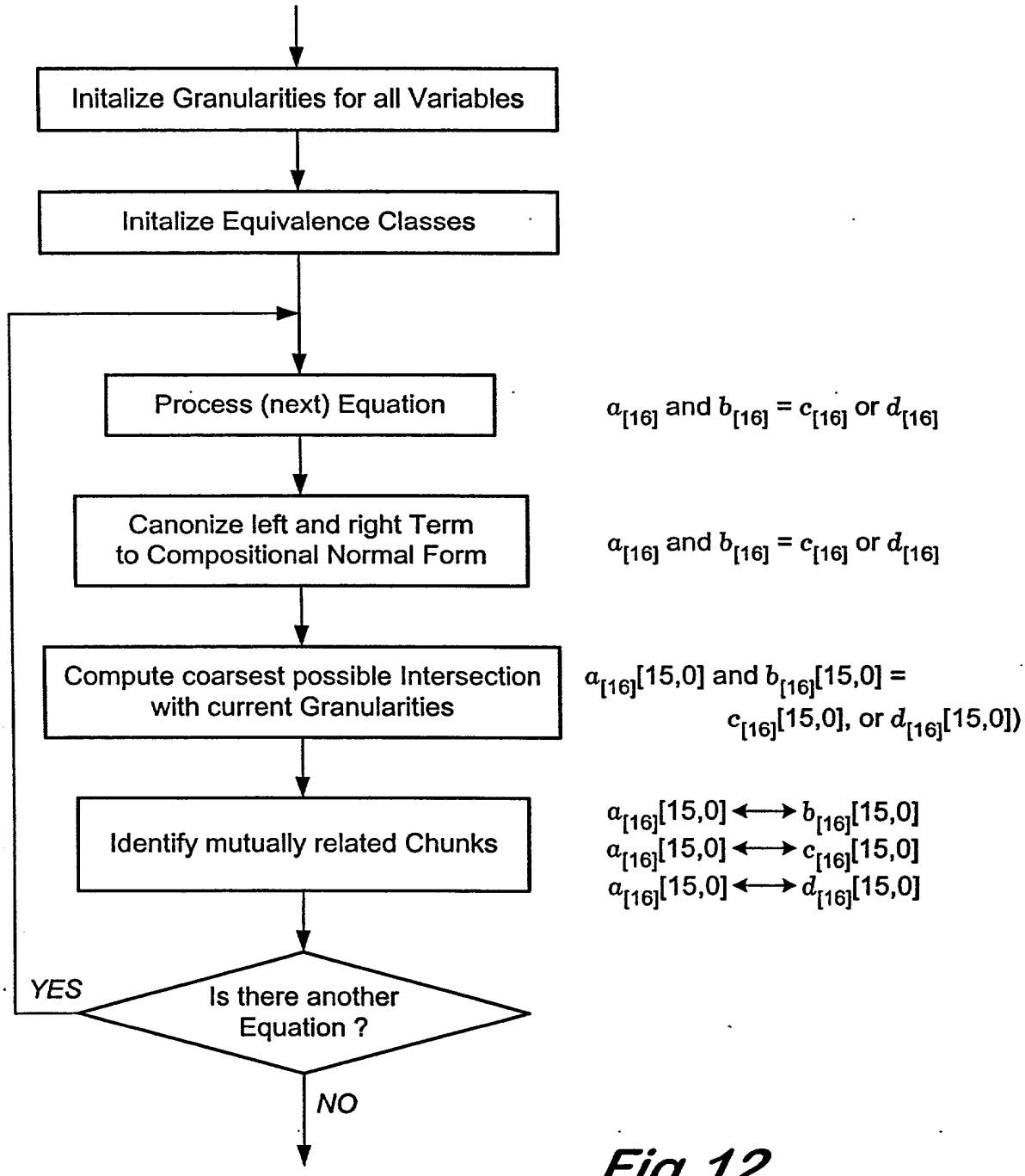
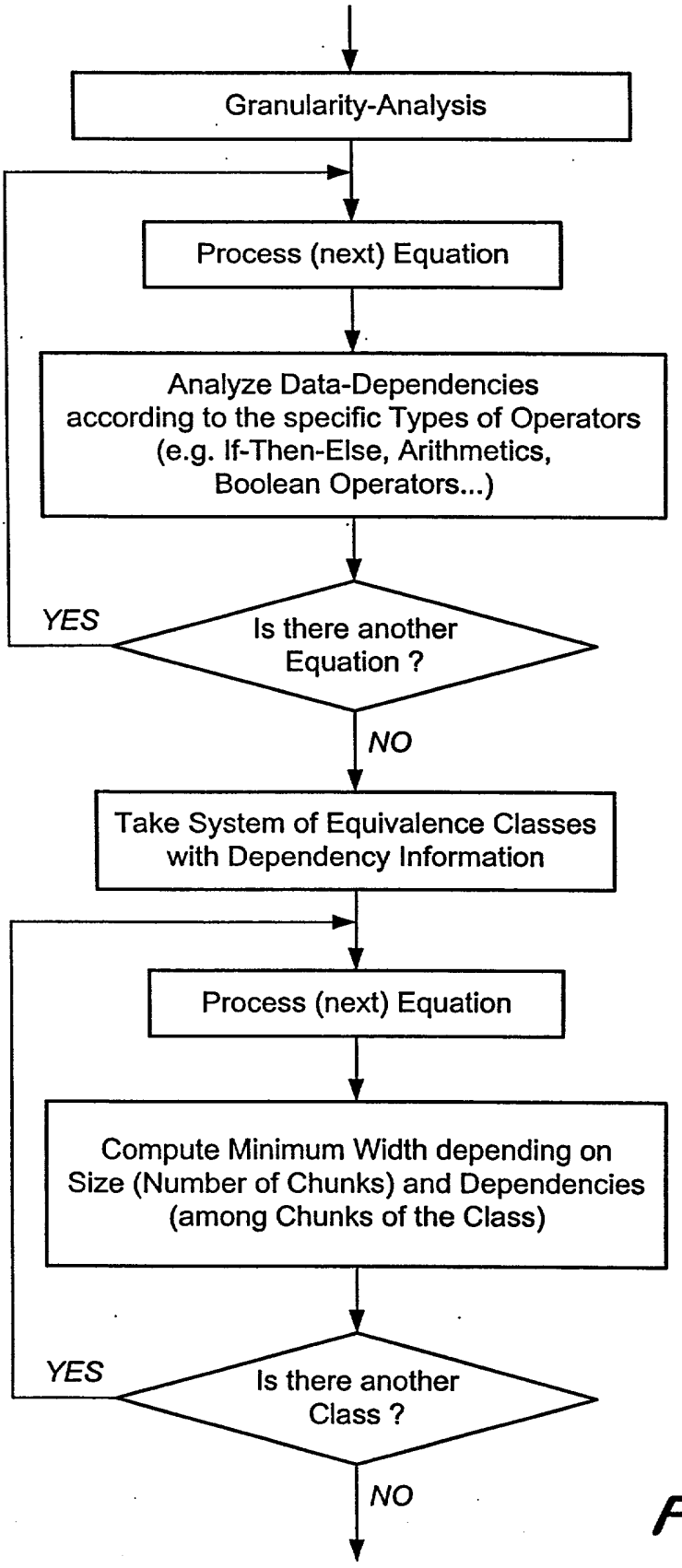


Fig. 12



$x_{[8]}[7,4] \otimes x_{[8]}[3,0] = a_{[4]}[3,0] \otimes b_{[4]}[3,0]$

Dependencies not conditioned by operator

C, Dependencies not conditioned by operator

$min = 1$

Fig. 13

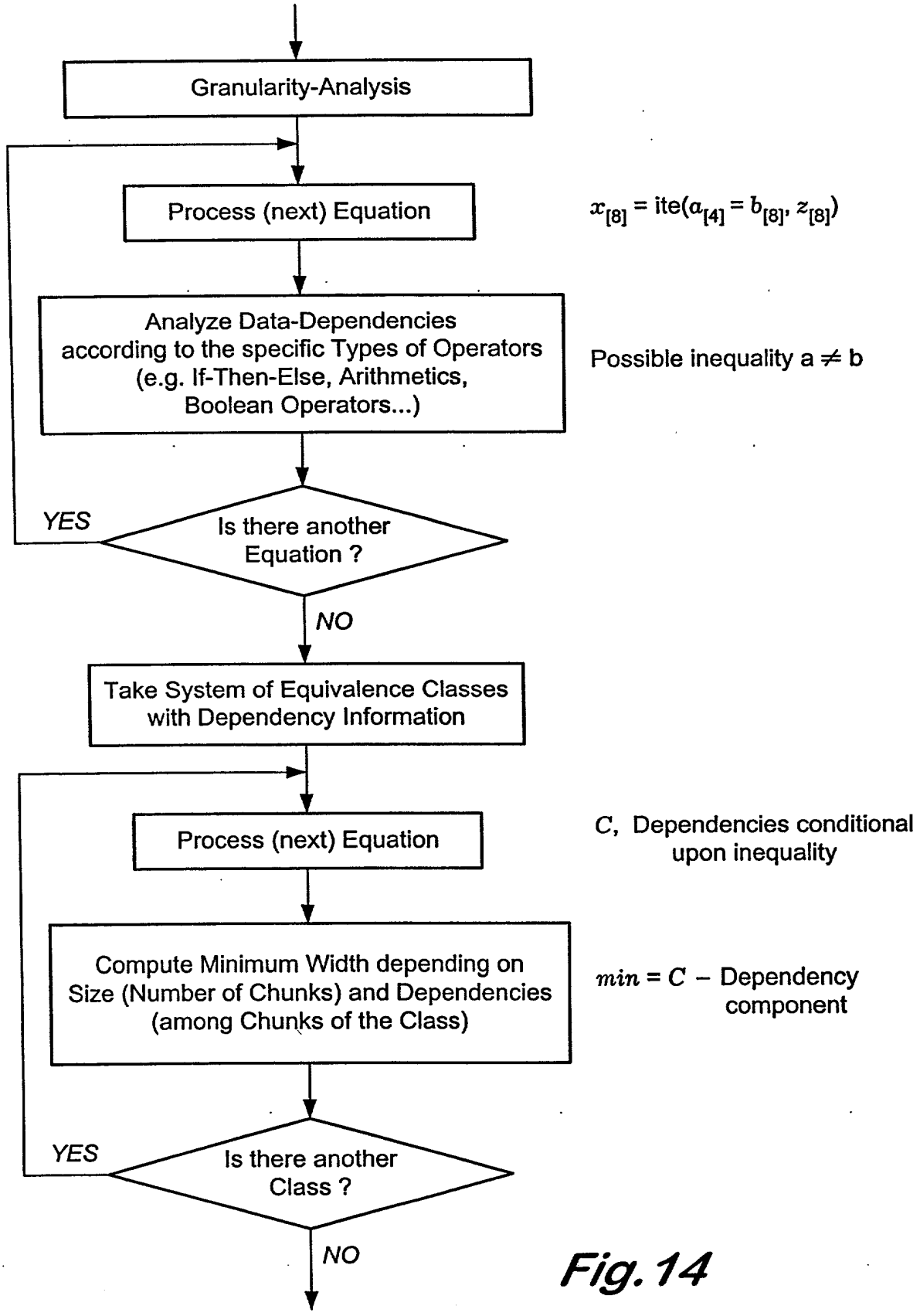


Fig. 14

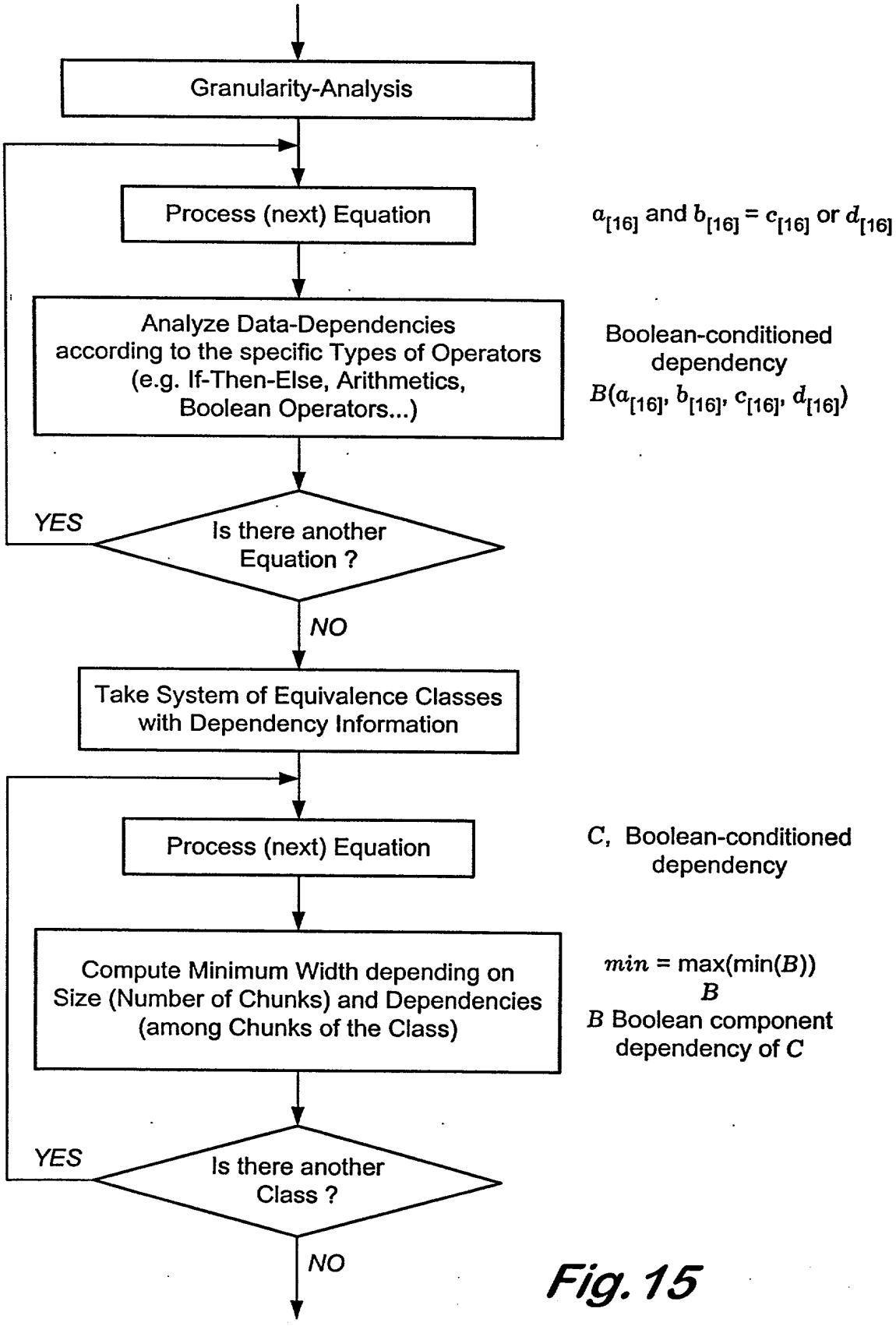


Fig. 15

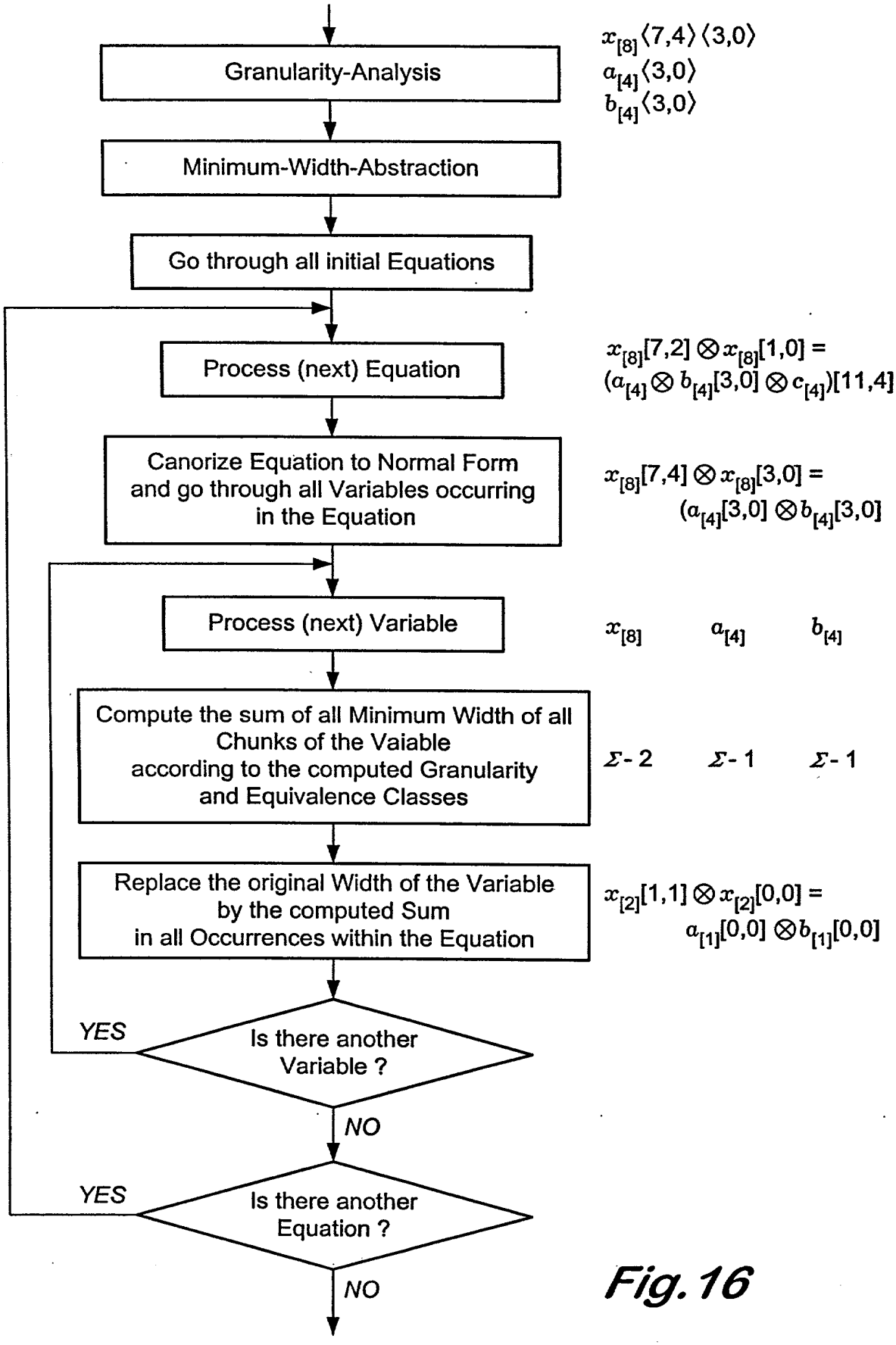


Fig. 16

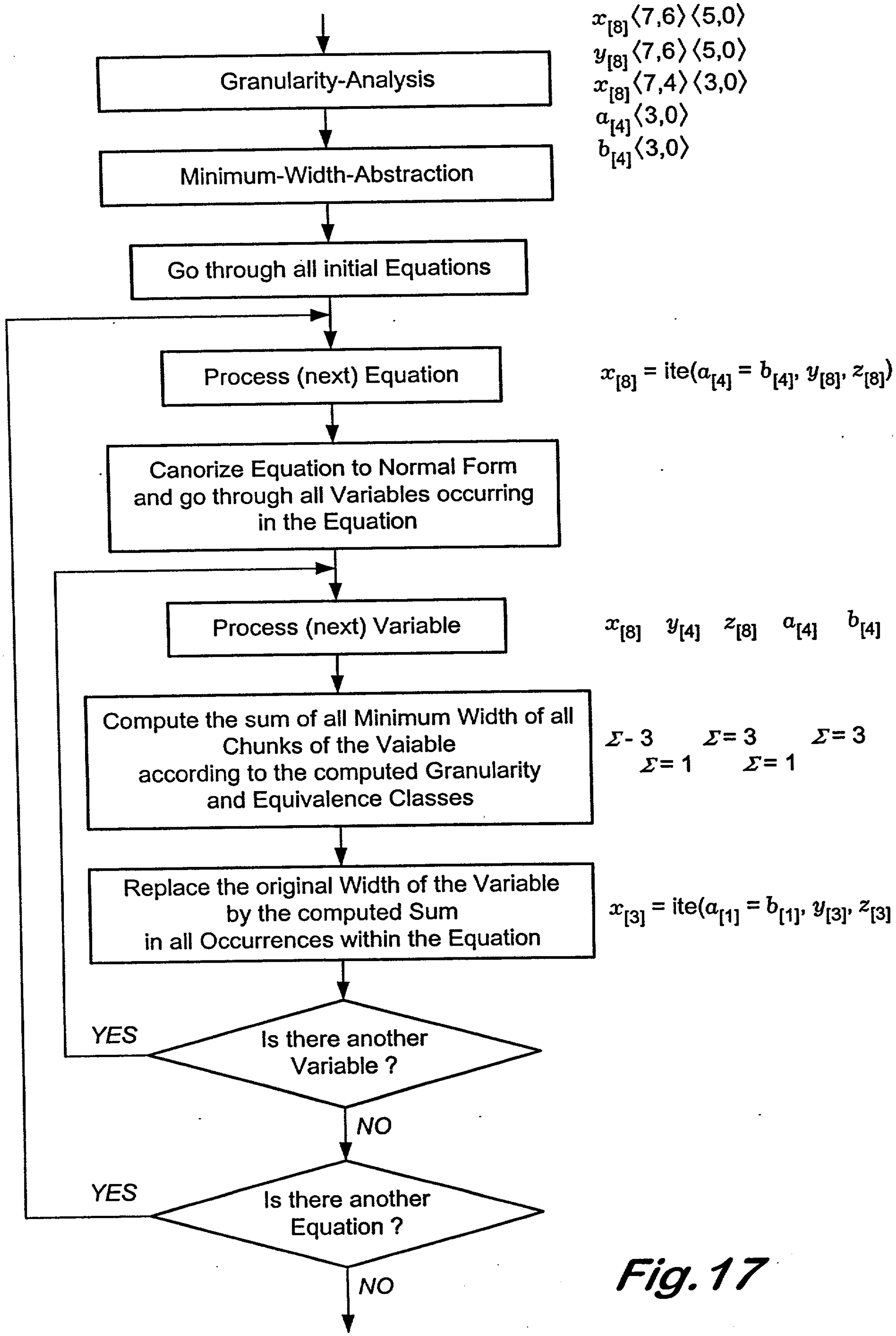

Granularity-Analysis
Minimum-Width-Abstraction
Go through all initial Equations
Process (next) Equation
Canorize Equation to Normal Form and go through all Variables occurring in the Equation
Process (next) Variable
Compute the sum of all Minimum Width of all Chunks of the Vaiable according to the computed Granularity and Equivalence Classes
Replace the original Width of the Variable by the computed Sum in all Occurrences within the Equation
Is there another Variable ?
YES
NO
Is there another Equation ?
YES
NO
$x_{[8]}\langle 7,6\rangle\langle 5,0\rangle$
$y_{[8]}\langle 7,6\rangle\langle 5,0\rangle$
$x_{[8]}\langle 7,4\rangle\langle 3,0\rangle$
$a_{[4]}\langle 3,0\rangle$
$b_{[4]}\langle 3,0\rangle$
$x_{[8]} = ite(a_{[4]} = b_{[4]}, y_{[8]}, z_{[8]})$
$x_{[8]}$ $y_{[4]}$ $z_{[8]}$ $a_{[4]}$ $b_{[4]}$
Σ- 3 Σ= 3 Σ= 3
Σ= 1 Σ= 1
$x_{[3]} = ite(a_{[1]} = b_{[1]}, y_{[3]}, z_{[3]})$

Fig. 17

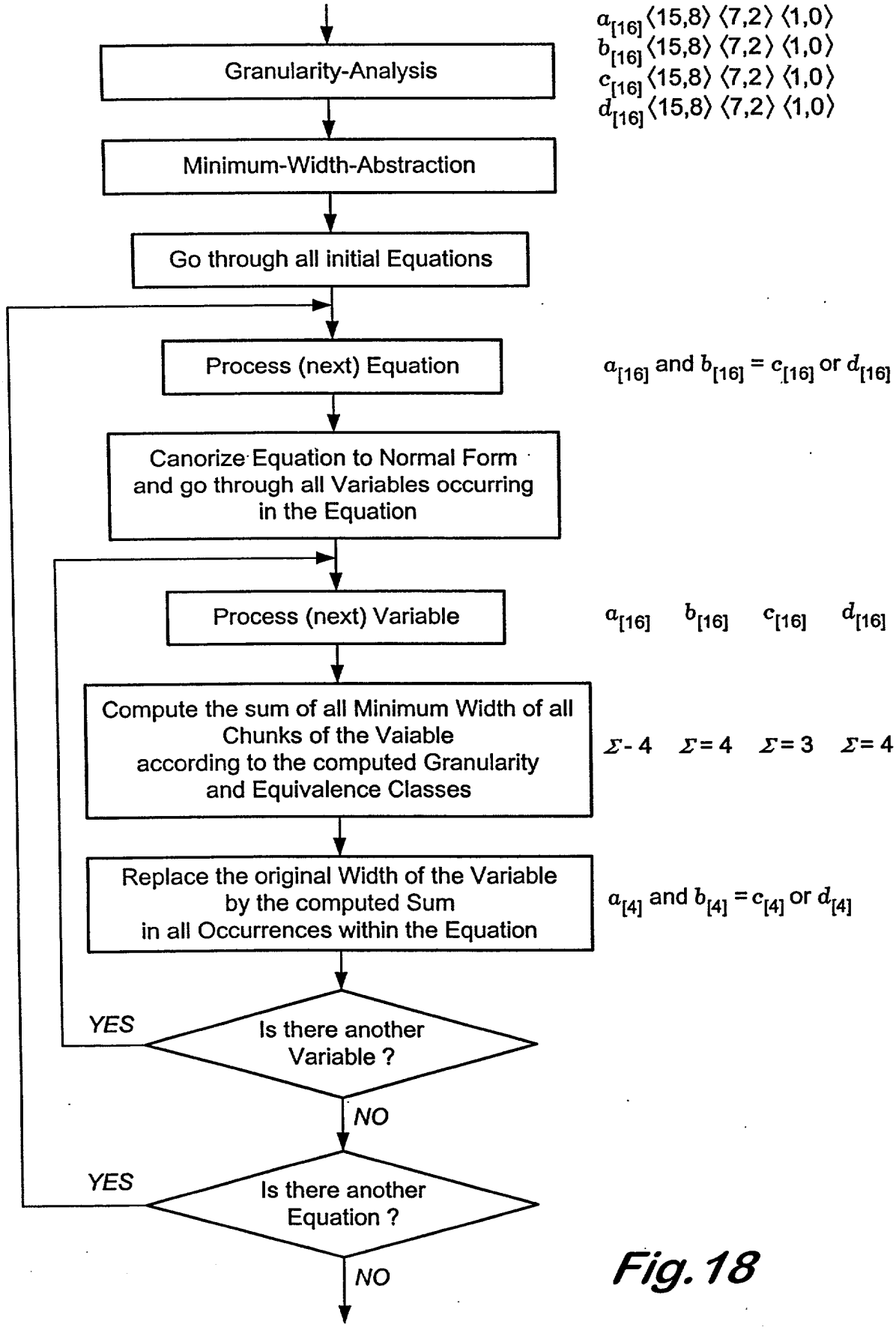


Fig. 18